I. Option Valuation:
Priced Factors in the Options Market

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Outline

- Risk and risk aversion
- The Black-Scholes world
- Something is/are missing (Coval and Shumway, 2001)
- A comprehensive investigation (Bakshi, Cao and Chen, 1997)
in economic theory, an individual is risk averse in the sense that
\( u(w) > Eu(w + \tilde{\epsilon}) \) for \( E(\tilde{\epsilon}) = 0 \) (a fair lottery) if and only if his von Neumann-Morgenstern utility function \( u \) is concave

a risk premium \( \pi \) is paid by a risk averse individual to avoid holding a risk \( \tilde{\epsilon} \)

\[
u(w - \pi) = Eu(w + \tilde{\epsilon})
\]

Taylor expansion around \( w \) gives

\[
u(w - \pi) \approx u(w) - \pi u'(w)
\]

\[
Eu(w + \tilde{\epsilon}) \approx E(\nu(w) + \tilde{\epsilon}u'(w) + \frac{1}{2}\tilde{\epsilon}^2 u''(w))
\]

\[
= u(w) + \frac{1}{2} \text{Var}(\tilde{\epsilon})u''(w)
\]

\[
\Rightarrow \pi = -\frac{1}{2} \frac{u''(w)}{u'(w)} \text{Var}(\tilde{\epsilon})
\]
Risk and risk premium

Asset prices respond positively or negatively to risks

- Risk↑ price↓ ⇒ negative $\beta$, expected returns↑, positive risk premium
  for example, bond price and interest rate risk, the positive risk premium are compensation for investors to undertake the risk

- Risk↑ price↑ ⇒ positive $\beta$, expected returns↓, negative risk premium
  for example, put option price and market risk, these assets offer positive returns in bad states of the world hence they require premium to hold them
Some notation

- underlying asset price: $S$
- asset returns: $R$
- strike price: $X$
- call option price: $c$
- put option price: $p$
- short-term riskfree interest rate: $r$
- asset returns volatility: $\sigma$
The Black-Scholes paradigm

- key assumptions
  - asset prices follow lognormal distribution
  - asset returns follow geometric Brownian motion
  - volatility of the asset returns $\sigma$ is constant

- implications
  - higher moments of asset returns are irrelevant: skewness, kurtosis
  - market risk is the only risk factor
  - options are redundant assets whose prices can be replicated; complete market
  - no risk premium for delta-netural positions, returns equal riskfree rate
Coval and Shumway (2001)

- focus on returns of call and put options and straddle positions
- options essentially take levered positions of the underlying asset
- call options are riskier than the underlying assets as measured by $\beta$

\[
\beta_c = \frac{S}{c} \mathcal{N}(d_1) \beta_S \Rightarrow \frac{\beta_c}{\beta_S} = \frac{\partial c/c}{\partial S/S} > 1
\]

- hence higher expected returns that increase in $X$
- put options have negative expected returns as their prices move negatively with underlying asset price
- hence negative expected returns that increase in $X$
- straddles are combinations of call and put options with the same strike and maturity
- zero-beta, or zero-delta, straddles typically hedge away market risk
- hence expected returns equal riskfree rate
- however, zero-beta straddles are exposed to market volatility
- zero-beta straddles have positive $\beta_\sigma$, they gain value when volatility ↑, hence insure against volatility risk
- if there is a risk premium for volatility risk, expected returns will be less than the riskfree rate
Coval and Shumway (2001)

- data
  1. weekly returns of European-style SP500 index options (SPX) between 1990.01 to 1995.10
  2. daily returns of American-style SP100 index options (OEX) between 1986.01 to 1995.12
- use simple returns not log returns
- this is common practice in dealing with low frequency data and are more appropriate for options
- for the index itself
  1. SP500: average weekly returns 0.18%, median weekly returns 0.20%, covers a fairly normal period of the market
  2. SP100: average daily returns 0.04%, median daily returns 0.05%, covers the market crash of Oct 1987
- grouped according to moneyness (K/S)
Table 1: call options

- SPX ATM option returns between 1.85% and 2.00%
- increasing (almost) monotonically with X
- not statistically significant
- BS $\beta$ very high and increasing with K
- given the $\beta$ estimates, returns should be much higher
- assume market risk premium of 6%, for ATM options should earn weekly returns between $6\% \times \frac{31.2}{52} = 3.60\%$ and $6\% \times \frac{40.02}{52} = 4.62$
- call options returns are too low; they should compensate holders for other risk(s)
Table 2: put options

- returns are all negative, statistically significant, and increase in $X$
- weekly ATM put option returns between $-9.50\%$ to $-7.71\%$
- highly negative BS $\beta$
- assume again market risk premium of 6%, ATM put returns should be around $-4.07\%$ to $-3.59\%$
- similar story that put option returns are too low to compensate only for market risk; other risk(s) are priced in options market
Table 3: zero-beta straddles

- weekly SPX straddle returns are negative and statistically significant from zero between $-4.49\%$ to $-2.89\%$
- this is true after controlling for measurement error in $\beta$ estimation, non-synchronous trading, different sample periods, and inclusion of market crash
- as straddles are exposed to volatility risk, the negative returns are evidence that volatility is priced in the options market with negative risk premium
Bakshi, Cao and Chen (1997)

- compare pricing accuracy and hedging effectiveness of a number of models
- features include stochastic volatility, random jumps, and stochastic interest rate
- use S&P 500 index options (European-style) from June 1988 to May 1991
- they find that stochastic volatility is of first-order importance, reducing BS pricing errors by 25% to 60%
- adding random jumps improves performance of short-term options while adding stochastic interest rate improves performance of long-term options
- hence evidence that volatility risk and jump risk are priced in index options
- stochastic interest rate is of paramount importance in pricing interest rate derivatives
the model is specified as follows (refer to the paper for notations),

\[
\frac{dS(t)}{S(t)} = [R(t) - \lambda \mu_J]dt + \sqrt{V(t)}d\omega_S(t) + J(t)dq(t)
\]

\[
dV(t) = [\theta_v - \kappa_v V(t)]dt + \sigma_v \sqrt{V(t)}d\omega_v(t)
\]

\[
\ln[1 + J(t)] \sim N(\ln[1 + \mu_J] - \frac{1}{2}\sigma_J^2, \sigma_J^2)
\]
To recap

- there are ample evidence that more risks than the market risk is priced in the options market
- one stylized fact is that option-implied volatilities exhibit volatility smile/smirk
- adding volatility risk and jump risk significantly improves performance of option pricing models